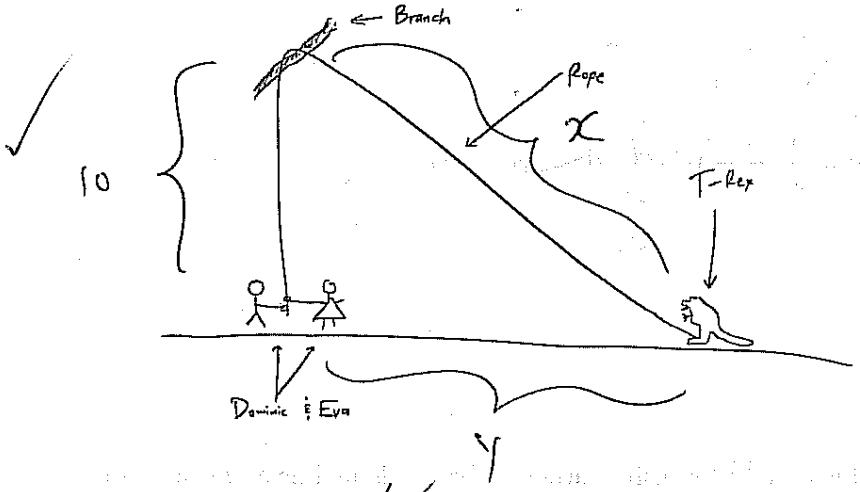
Quiz 4A, Calculus I - Calculators okay

Dr. Graham-Squire, Spring 2014

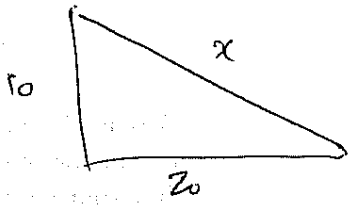
Name: Key

1. (4 points) Dominic and Eva have thrown a rope over a tree branch that is 10 feet high. They tied one end of the rope to a robotic dinosaur named Jagger the T-Rex, and they are pulling the other end of the rope so that Jagger is dragging along the ground. If they pull in the rope at 2 ft/sec, how fast is Jagger being dragged forward when he is 20 feet away from them? Show your work to receive full points. Round to nearest 0.01.



$\frac{dx}{dt} = -2$ ft/sec

WTF? $\left.\frac{dy}{dt}\right|_{y=20}$



$\frac{d}{dt}\left(x^2 = 10^2 + y^2\right)$

$x = \sqrt{10^2 + 20^2}$

$x = \sqrt{500} = 22.36$

$2x\frac{dx}{dt} = 2y\frac{dy}{dt}$

$\frac{2(22.36)(-2)}{2(20)} = \frac{2(20)\frac{dy}{dt}}{2(20)}$

$-2.24 \text{ ft/sec} = \frac{dy}{dt}$

Jagger is being dragged at a rate of 2.24 ft/sec

2. (3 points) Use calculus to find the absolute maximum and minimum of $f(x) = \frac{4x}{x^2+9}$ on the interval [-1,5]. You can check your work with a graphing calculator, but you must show your work with calculus to receive full points.

$f'(x) = \frac{4(x^2+9) - 4x(2x)}{(x^2+9)^2} = \frac{4x^2 + 36 - 8x^2}{(x^2+9)^2} = \frac{-4x^2+36}{(x^2+9)^2}$ ✓

$f'(x) = 0$ when $-4x^2 + 36 = 0 \Rightarrow x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3$

$x = -3$ not in domain $[-1,5]$, can ignore. ✓

check $f(3) = \frac{12}{18} = \boxed{\frac{2}{3} = 0.67}$ → abs. max

$f(-1) = \frac{-4}{10} = \boxed{-0.4}$ → abs. min

$f(5) = \frac{20}{34} = 0.588$ ✓

3. (3 points) Approximate the value of $\sqrt{0.9}$ by using either differentials or linearization. You should not need a calculator to find your approximation, but you can use a calculator to check that your answer is close to the real value.

$\sqrt{0.9} \approx \sqrt{1} = 1$ Let $p = 1$ | $dy = f'(s)\,dx$

$f(x) = \sqrt{x} \Rightarrow f'(x) = \frac{1}{2}x^{-1/2}$ | $f(x) = \sqrt{x}$

$L(x) = f(1) + f'(1)(x-1)$ | $s = 1$

$L(x) = 1 + \frac{1}{2}(x-1)$ | $dx = -0.1$

$L(x) = \frac{1}{2}x + \frac{1}{2}$ | $dy = \frac{1}{2}(1)^{-1/2} \cdot (-0.1) = (0.5)(-0.1)$

$L(0.9) = \frac{1}{2}(0.9) + 0.5$ | $dy = -0.05$

$= 0.45 + 0.5$

$= \boxed{0.95}$ | $\Rightarrow \sqrt{0.9} \approx 1 - 0.05 = \boxed{0.95}$